Exercise 4

The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

Solution

The area of a rectangle is its length l times its width w.

$$A = lw$$

Differentiate both sides with respect to t, using the product rule on the right side.

$$\frac{d}{dt}(A) = \frac{d}{dt}(lw)$$

$$\frac{dA}{dt} = \frac{dl}{dt}w + l\frac{dw}{dt}$$

The length is increasing by 8 centimeters per second, so $dl/dt = 8$ cm/s. The width is increasing by 3 centimeters per second, so $dw/dt = 3$ cm/s. Therefore, when the length is 20 cm and the width is 10 cm, the rate that area is increasing is

$$\left.\frac{dA}{dt}\right|_{\substack{l=20 \\ w=10}} = (8)(10) + (20)(3) = 140\ \frac{\text{cm}^2}{\text{s}}.$$